3-D edge singularities in elastic domains

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This talk summarizes a long journey with Monique towards an explicit asymptotic representation of the solution to linear elasticity in the vicinity of a straight or curved edge in a three-dimensional domain. It started in 2000 during one of my visits in Rennes, following my question whether she can write down such an explicit asymptotic series. Finally, in [1] a theoretical framework was presented allowing the displacement field to be expressed as:

$$\boldsymbol{u}(r,\theta,z) = \sum_{i=1} \sum_{j=0} \partial_z^j A_i(z) r^{\alpha_i+j} \phi_{ij}(\theta) \tag{1}$$

The displacements (and therefore the stresses) are represented by a family of eigenfunctions identical to 2-D plane strain ones, complemented by *shadow singularities*, and associated edge stress intensity functions $A_i(z)$, which are functions along the straight edge. Having the explicit solution, the quasi-dual function method (QDFM) was introduced for the extraction of edge stress intensity functions (ESIFs). It has two important advantages: a) The ESIFs may be extracted away from the singular edge, thus avoiding the need for a refined FE mesh in the vicinity of the singular edge, b) ESIFs are obtained as functions along the edge and not as pointwise values.
The theoretical framework has been very successfully applied to anisotropic material, composites and bi-material interfaces and numerical results have been presented [2, 3, 4].
For curved singular edges, the asymptotic expansion (1) is further expanded by an additional sum [5, 6]:

$$\boldsymbol{u}(\rho,\varphi,\theta) = \sum_{i=1} \sum_{j=0} \partial_\theta^j A_i(\theta) \rho^{\alpha_i} \sum_{k=0} \left(\frac{\rho}{R}\right)^{j+k} \phi_{ijk}(\varphi) \tag{2}$$

Lately, we have extended the QDFM to extract T-stresses, associated with the integer eigenvalue.
In this talk we present the machinery for the systematic derivation of the eigenpairs and their shadows, followed by the use of the QDFM for the extraction of ESIFs from high-order finite element methods. Numerical examples are provided that demonstrate the efficiency, robustness and high accuracy of the proposed QDFM.

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